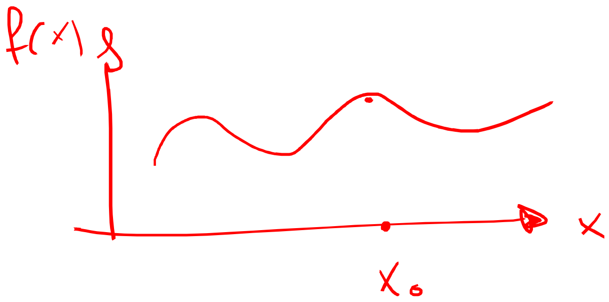
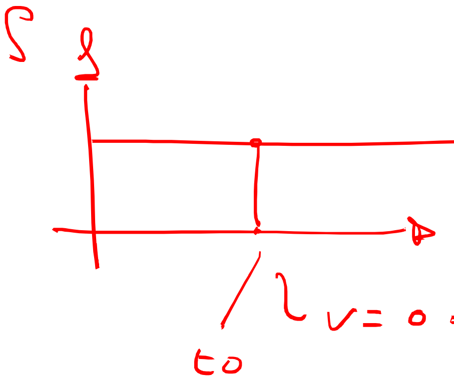
DERIVATA



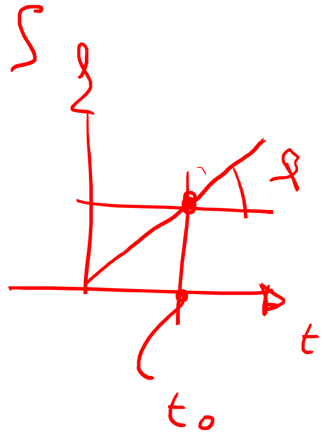
$$\lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x} = \left.\frac{df}{dx}\right|_{x=x_0}$$

$$v = \frac{ds}{dt}$$

esempio



$v = 0 = \frac{ds}{ds}$

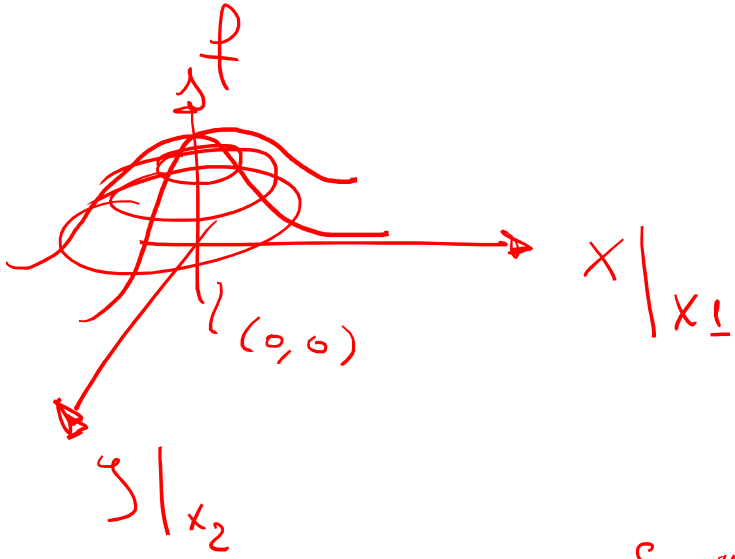


$v = \frac{ds}{dt} = t$

$\frac{d\beta s}{dt}$

$= \beta$

$f(x)$

$f(x,y) \rightarrow$ 

$\downarrow$

$$\frac{df}{dx}$$

• Secondo il Rapporto incrementale.

$$\frac{\partial f}{\partial x} = \left.\frac{df}{dx}\right|_{y=\mathrm{const}} \; ; \quad \frac{\partial f}{\partial y} = \left.\frac{df}{dy}\right|_{x=\mathrm{const}} \; ;$$

- Derivata
- Derivata parziale $\Rightarrow$

1. Gradiente $\to \nabla f$
2. Divergenza $\to \nabla \cdot \bar{F}$
3. $\nabla \times f$ - Rotore
   $\nabla \wedge \bar{f}$

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1. Gradiente

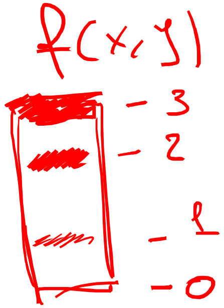
operatore $\to$ $\nabla f \to$ campo vettoriale

f scalare

$\text{grad}(f)$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) f(x,y,z) \xrightarrow{3D} \left(\frac{\partial}{\partial x} f, \frac{\partial}{\partial y} f, \frac{\partial}{\partial z} f\right)$$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) f(x,y) \xrightarrow[2D]{} \left(\frac{\partial}{\partial x} f, \frac{\partial}{\partial y} f\right)$$



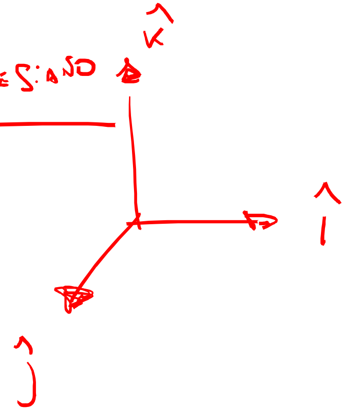
$$f(x,y)=c_1+c_2 x$$

$$\nabla f = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)(c_1+c_2 x)$$

$$= \left(\frac{\partial}{\partial x}(c_1+c_2 x), \frac{\partial}{\partial y}(c_1+c_2 x)\right) =$$

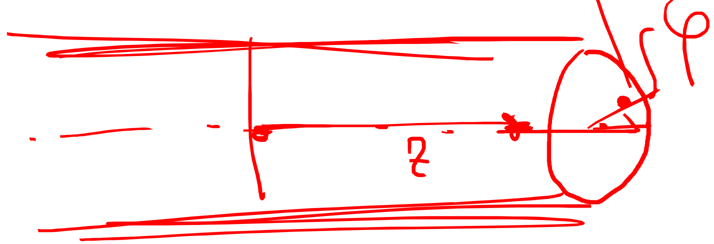
$$= (c_2, 0) = \text{campo vettoriale} \equiv \nabla f$$

Cartesiano



$$\nabla f = \frac{\partial}{\partial x} f\, \hat{i} + \frac{\partial}{\partial y} f\, \hat{j} + \frac{\partial}{\partial z} f\, \hat{k}$$

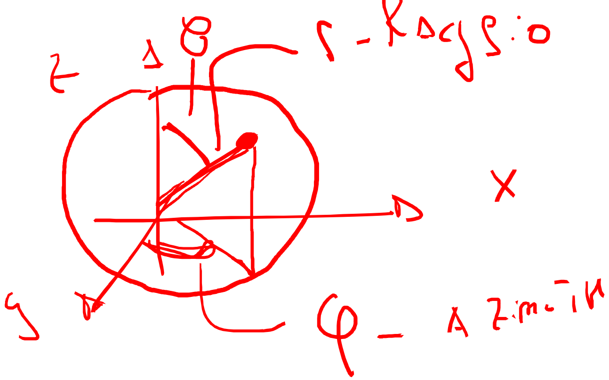
Cilindrico



$$\nabla f = \frac{\partial f}{\partial \rho} \hat{e}_\rho + \frac{1}{\rho} \frac{\partial f}{\partial \varphi} \hat{e}_\varphi + \frac{\partial f}{\partial z} \hat{e}_z$$

$\hat{e}_\rho$, $\hat{e}_\varphi$, $\hat{e}_z$ | versori $\Rightarrow$ $\hat{i}$, $\hat{j}$, $\hat{k}$

Sferico, polare



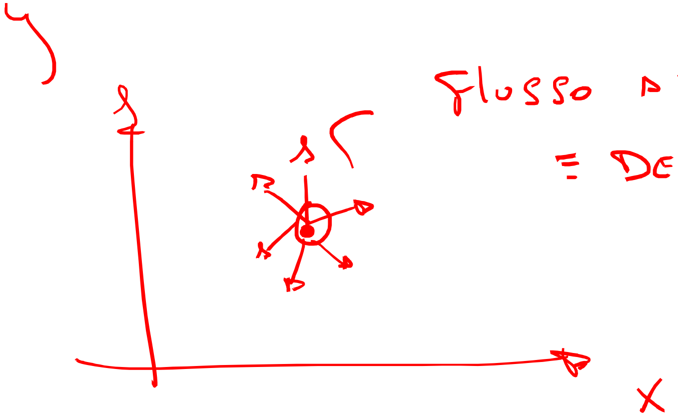
$$\nabla f(r, \theta, \varphi) = \underbrace{\frac{\partial f}{\partial r} \hat{e}_r}_{\text{versore}} + \underbrace{\frac{1}{r} \frac{\partial f}{\partial \theta} \hat{e}_\theta}_{\text{versore}} + \frac{1}{r \sin\theta} \frac{\partial f}{\partial \varphi} \underbrace{\hat{e}_\varphi}_{\text{versore}}$$

2) Divergenza → campo vettoriale ⇒ campo scalare

$$\nabla \cdot \bar{F} \to \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \dots\right) \cdot \left(F_x, F_y, \dots\right)$$

prodotto scalare

$$2D \to \left(\frac{\partial}{\partial x} F_x + \frac{\partial}{\partial y} F_y\right)$$



flusso attraverso una superficie infinitesimamente piccola

= densità spaziale del flusso ⇒ $\nabla \cdot \bar{F}$

flusso

- Derivata
- Derivata parziale
- Gradiente → ∇f → direzione di massimo cambiamento di una f scalare
- Divergenza → $\nabla \cdot \bar{F}$ → densità di flusso del campo
- Rotore → $\nabla \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$

$\nabla \times \bar{F} = 0$

$\bar{F}$ é irrotazionale

$\nabla \times \bar{F}$ → indica la rotazione puntuale/locale del campo vettoriale $\bar{F}$

Derivata totale

$f(x, y) \qquad x \to x(t)$

$\qquad\qquad\quad\;\, y \to y(t)$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial t}\cancel{\frac{dt}{dt}}^{\,1}$$

$$Df \rightarrow \frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial t}$$

$$\frac{Df}{Dt} \qquad f \equiv v \Rightarrow \frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} v_x + \frac{\partial v}{\partial y} v_y$$

$$\rightarrow a \quad = \frac{\partial v}{\partial t} + \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) v \cdot (v_x, v_y)$$

$$= \partial v / \partial t + \nabla \cdot v \bar{V}$$

* CAMPO → VETTORIALE (FORZE)

↓

SCALARE

POTENZIALE → ENERGIA CHE UN CORPO HA ALL'INTERNO DEL CAMPO DI FORZE A CAUSA DELLA POSIZIONE DEL CORPO NEL CAMPO

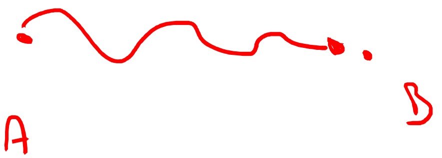
$L_{1-2} = U_1 - U_2$

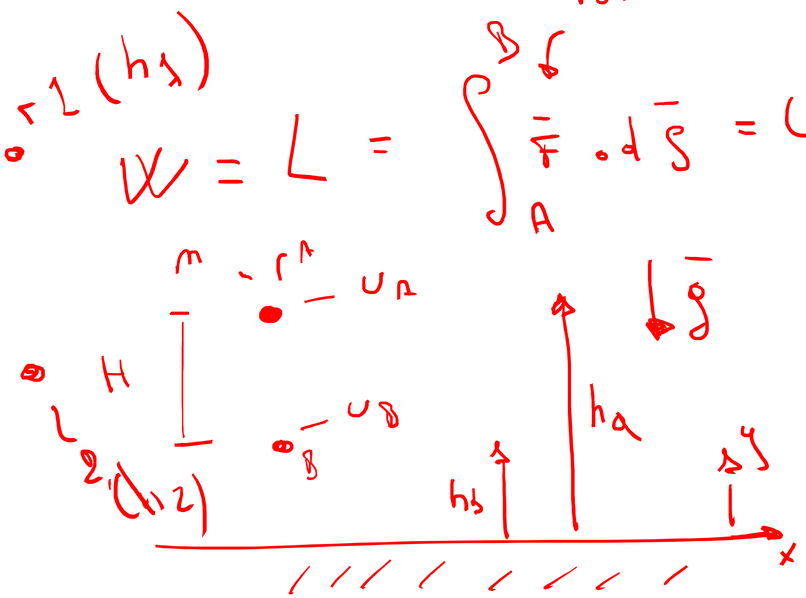
↓

$mgh_1 - mgh_2$

FORZA DEL CAMPO

$$W = L = \int_A^B \bar{F} \cdot d\bar{s} = U_A - U_B = -(U_B - U_A) = -\nabla U$$





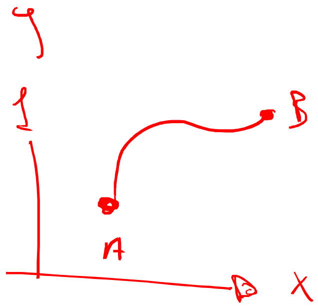
L_{AB}

$W_{AB} = mgH = U_A - U_B$

$U_B(h_b = 0) = 0$ ⟹ $W_{AB} = mgh_a = U_A$

$$W = F\,S$$

$$= \int_A^B \bar{F} \cdot d\bar{S} \rightarrow [\bar{J}]$$



$$W\Big|_{A-B} = U_A - U_B$$

$$= -(U_B - U_A)$$

$$W\Big|_{A-B} = -\Delta U$$

$$A \rightarrow U_A, \quad B \rightarrow U_B \Rightarrow U_A \equiv U_B \Rightarrow W_{A-B} = -\Delta U = -(U_B - U_A) = 0$$

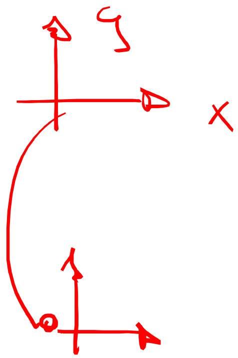
$$\rightarrow \oint_{\partial S} \bar{F} \cdot d\bar{S} = 0 \qquad \int_S \nabla \times \bar{F} = 0 \Rightarrow \nabla \times \bar{F} = 0$$

$$\forall\, \partial S,\ S$$

$$\nabla \times \bar{F} = 0 \Rightarrow \bar{F} = -\nabla U$$

$-m\bar{g} = \bar{F}$

$$U = mgh$$
$$= mgy$$



$$\bar{F} = -\nabla \overset{\int mgy}{U}$$
$$= -(\partial_x, \partial_y)\, U$$
$$= -(0, mg) \Rightarrow \bar{F}(mgh) = (0, -mg)$$

$-|m\bar{g}|$

- T → energia cinetica

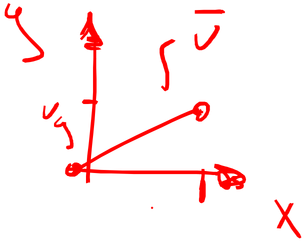
- U → energia potenziale

$$L(q,\dot q) = T(\dot q) - U(q)$$

→ $$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_i}\right) = 0$$ → equazioni di eulero-Lagrange

→ permettono di derivare le equazioni del moto;

- Scalare → numero → Tensore di ordine 0
- Vettori →



$\bar v \equiv (v_x, v_y)$

$\hat i = (1,0)$

$\hat j = (0,1)$

- Tensori →

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$B - b_j$

a_{ij}

$$A \cdot B = a_{ij} \cdot b_j$$

$$a_{ij} b_j = \begin{pmatrix} a_{1j} b_j \\ a_{2j} b_j \end{pmatrix} = \begin{pmatrix} a_{11} b_1 + a_{12} b_2 + a_{13} b_3 \\ a_{21} b_1 + a_{22} b_2 + a_{23} b_3 \end{pmatrix} \begin{matrix} \text{Riga 1} \\ \text{Riga 2} \end{matrix}$$

$x = (x_1, x_2, x_3) \to x_i$

$y = (y_1, y_2, y_3) \to y_i$

$x_i y_i = (x_1 y_1 + x_2 y_2 + x_3 y_3)$

$c_i \to \bar{c}$

d_i

- $c_i d_i$
- $c_{ij} d_j$
- $c_{ij} d_{ij}$

- $$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$$\varepsilon_{ijk} = \begin{cases} 1 & 123 \quad 231 \quad 312 \\ -1 & 321 \quad 132 \quad 213 \\ 0 & i = j \\ & i = k \\ & j = k \end{cases}$$