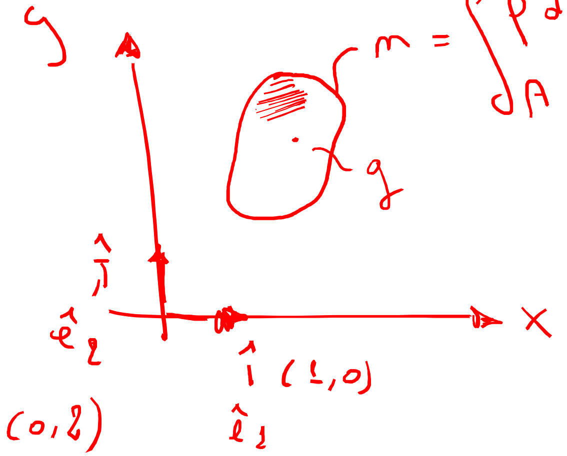
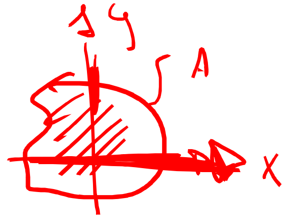


$$m = \int_A \rho \, dA$$

$$x_g = \frac{1}{m} \int_A \rho \, x \, dA \rightarrow S_y$$

$$y_g = \frac{1}{m} \int_A \rho \, y \, dA$$



- momenti statici:

$$S_x = \int_A \rho \cdot y \cdot dA$$

$$S_y = \int_A \rho \cdot x \cdot dA$$

$$g \begin{cases} x_g = S_y / m \\ y_g = S_x / m \end{cases} \Rightarrow \begin{cases} S_y = x_g \cdot m \\ S_x = y_g \cdot m \end{cases}$$

momenti del 1° ordine

$$G\begin{cases} x_g = \dfrac{1}{m}\displaystyle\int_A \rho x \, dA \\ y_g = \dfrac{1}{m}\displaystyle\int_A \rho y \, dA \end{cases} \Rightarrow \quad \rho = cost$$

$$m = A \cdot \rho$$

$$\begin{cases} x_g = \dfrac{1}{A \cdot \cancel{\rho}} \cdot \cancel{\rho} \displaystyle\int_A x \, dA \\ y_g = \dfrac{1}{A \cdot \cancel{\rho}} \cdot \cancel{\rho} \displaystyle\int_A y \cdot dA \end{cases} \quad \Big| \; \rho = cost$$

$$\begin{cases} x_g = \dfrac{1}{A}\displaystyle\int_A x \, dA \\ y_g = \dfrac{1}{A}\displaystyle\int_A y \, dA \end{cases}$$

$$S_x = \int_A y \overset{1}{\rho} \, dA = \int_A y \, dA$$

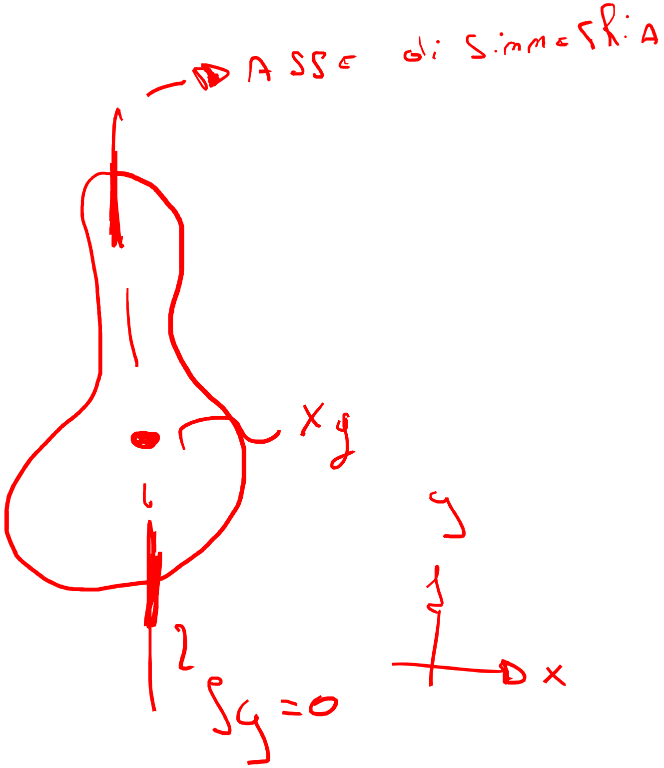
$$S_y = \int_A x \overset{1}{\rho} \, dA = \int_A x \, dA$$

$$\Rightarrow \begin{cases} x_g = S_y / A \\ y_g = S_x / A \end{cases}$$

$$\begin{cases} X_g = \frac{1}{A}\int_A x\,dA = S_y / A \\ y_g = \frac{1}{A}\int_A y\,dA = S_x / A \end{cases}$$

→ COORDINATE DEL BARICENTRO

– CENTRO DI MASSA –

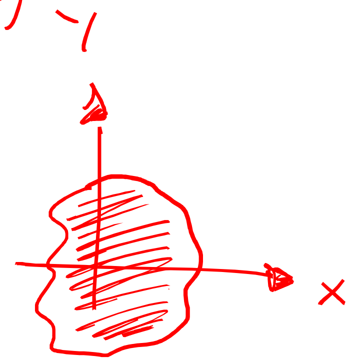


- BARICENTRO
- momenti statici (1° ordine)

$$S_x = \int_A y \, dA \qquad \rho = 1$$

$$S_y = \int_A x \, dA$$

- momenti del 2° ordine (momenti di inerzia)



$$I_x = \int_A y^2 \, dA$$

$$I_y = \int_A x^2 \, dA$$

$$I_{xy} = \int_A x \cdot y \, dA \rightarrow \text{centrifugo}$$

$$I_0 = \int_A \underbrace{(x^2 + y^2)}_{\rho^2} \, dA \rightarrow \text{polare}$$

- Raggi di inerzia:

$$r_x = \sqrt{I_x / A}$$

$$r_y = \sqrt{I_y / A}$$

$$r_o = \sqrt{I_o / A}$$

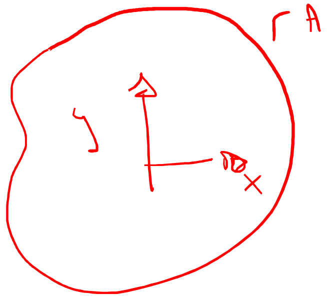
- $I_o = \int_A (x^2 + y^2)\, dA$

$= \int_A x^2 dA + \int_A y^2 dA$

$= I_y + I_x$

$\Rightarrow \boxed{I_o = I_x + I_y}$

$$\boxed{r_o^2 = r_x^2 + r_y^2}$$



2D

$S_x = \int_A y \, dA$

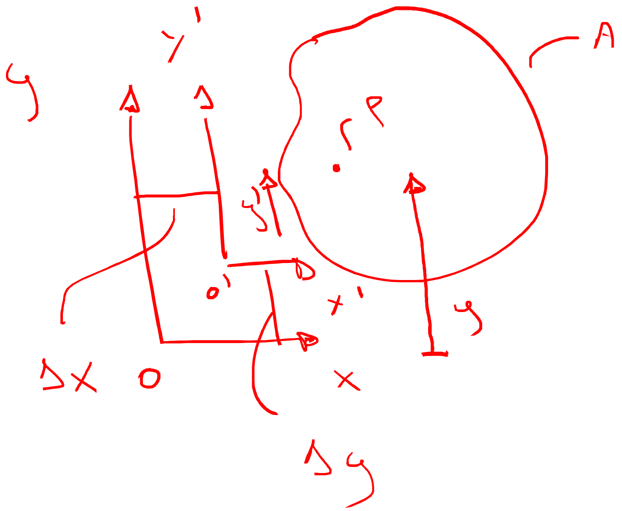
$S_y -$

$I_x = \int_A y^2 \, dA$

I_y ,

I_{xy} ,

I_o ,



$$I_x = \int_A y^2 \, dA$$

$P(x, y) \rightarrow 1°$

$P(x', y') \rightarrow 2°$

$$y_P = y'_P + \Delta y$$

$$I_x = \int_A y^2 \, dA = \int_A (y' + \Delta y)^2 \, dA = \int_A \left(y'^2 + \Delta y^2 + 2 y' \Delta y\right) dA$$

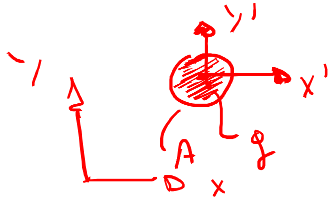
$$I_x = \int_A y'^2 \, dA + \int_A \Delta y^2 \, dA + 2 \int_A y' \Delta y \, dA$$

$$I_x = I_{x'} + \Delta y^2 A + 2 \Delta y \underbrace{\int_A y' \, dA}_{S_{x'}} \Rightarrow I_x = I_{x'} + \Delta y^2 A + 2 \Delta y \underbrace{S_{x'}}_{0}$$

$$I_x = I_{x'} + \Delta y^2 A$$

Lo baricentrico

• $I_x = I_x' + \Delta y^2 \cdot A$



$I_y = I_y' + \Delta x^2 \cdot A$

$I_{xy} = I'_{xy} + \Delta x \Delta y \cdot A$

$I_0 = I_0' + \Delta^2 \cdot A$

$\Delta^2 = (\Delta x^2 + \Delta y^2)$

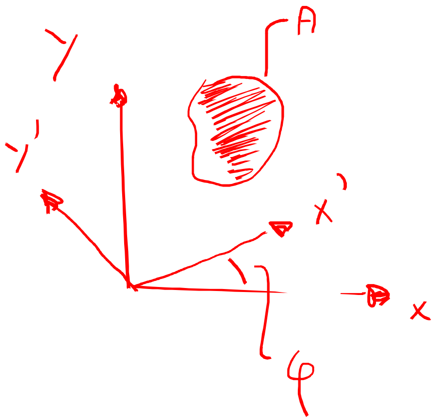
• x'y' → baricentrico

▪ Trasformazioni x traslazione

- Trasformazioni x rotazione

I_x

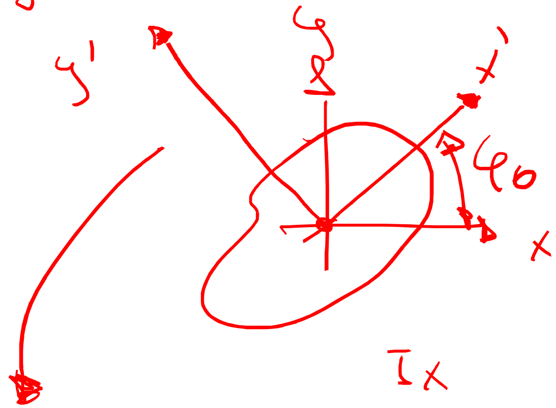
I_y ...



$$\begin{cases} I_{x'} = I_x \cos^2\varphi + I_y \sin^2\varphi - I_{xy} \sin 2\varphi \\ I_{y'} = I_x \sin^2\varphi + I_y \cos\varphi + I_{xy} \sin 2\varphi \\ I_{x'y'} = \dfrac{I_x - I_y}{2} \sin 2\varphi + I_{xy} \cos 2\varphi \end{cases}$$

$$\varphi_0 : I_{x'y'} = 0$$

$$\tan(2\varphi_0) = \frac{2 I_{xy}}{I_y - I_x}$$

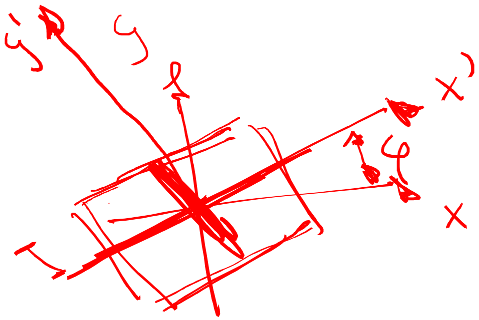


Assi principali di inerzia

I_x

I_y

I_{xy}



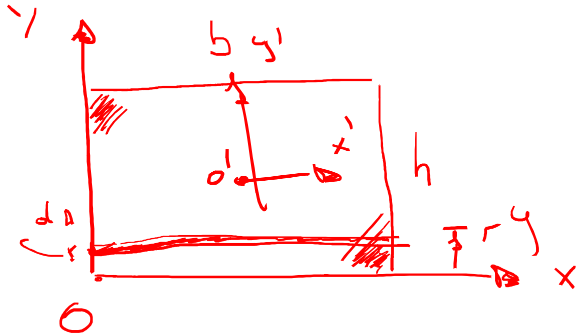
$$\begin{cases} I_{x'} - \min \\ I_{y'} - \max \end{cases}$$

φ_0 : $I_{x'} - \min$

$I_{y'} - \max$

$I_{xy'} - 0$

(Individua gli assi principali di inerzia)



$$\bar{I}_x = \int_A y^2 \, dA$$

$$\bar{I}_y$$

$$I_x = \int_A y^2 \, dA = \int_A y^2 b \, dy = b \int_A y^2 dy = b \int_0^h y^2 dy = b \frac{y^3}{3} \Big|_0^h$$

- $\bar{I}_x = \frac{b \cdot h^3}{3}$

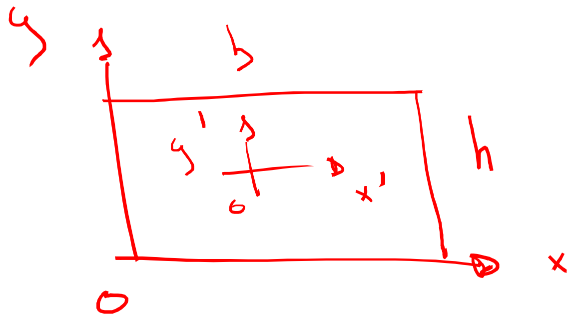
- $\bar{I}_y = \frac{h \cdot b^3}{3}$

$$\bar{I}_x = \bar{I}_{x'} + A \Delta y^2 \Rightarrow \frac{b h^3}{3} = \bar{I}_{x'} + (b \cdot h) \frac{h^2}{4}$$

$\bar{I}_x$ ↓ NON baricentrico

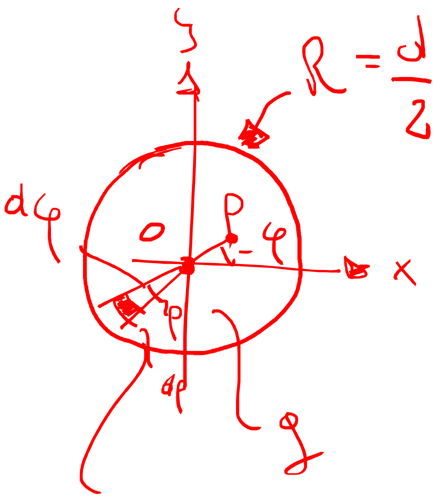
$\bar{I}_{x'}$ ↓ baricentrico

$$\Rightarrow I_{x'} = b h^3 \left(\frac{1}{3} - \frac{1}{4} \right) = b h^3 \frac{1}{12}$$



- $I_{x'} = \dfrac{b h^3}{12}$

- $I_{y'} = \dfrac{h \cdot b^3}{12}$



$dA = \rho \, d\rho \, d\varphi$

$$I_o = \int_A \rho^2 \, dA$$

ρ, φ

$\cancel{x, y}$

$$\int_A (x^2+y^2)\,dA = \underbrace{\int_A x^2\,dA}_{I_y} + \underbrace{\int_A y^2\,dA}_{I_x}$$

I_o

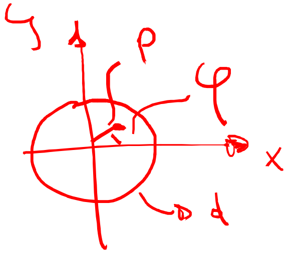
$$I_o = I_x + I_y$$

$$I_o = \int_A \rho^2 \rho \, d\rho \, d\varphi$$

$$I_o = \int_0^{2\pi} d\varphi \int_0^R \rho^3 \, d\rho = 2\pi \frac{R^4}{4} = \frac{\pi R^4}{2}$$

$$I_o = \frac{\pi R^4}{2} = \frac{\pi (d/2)^4}{2} = \frac{\pi d^4}{32}$$

$$I_x = \frac{I_o}{2} = I_y = \frac{\pi d^4}{64}$$



$$I_x = I_y = \frac{\pi d^4}{64}$$

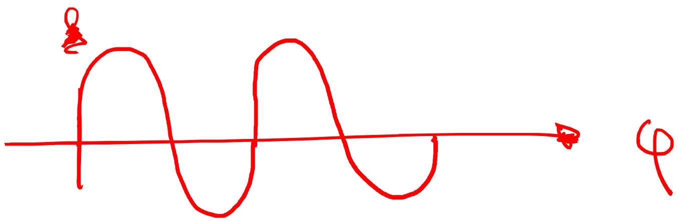
$$I_{xy} = \int_A x y \, dA$$

$$= \int_A \rho^2 \cos\varphi \, \sin\varphi \, \rho \, d\rho \, d\varphi$$

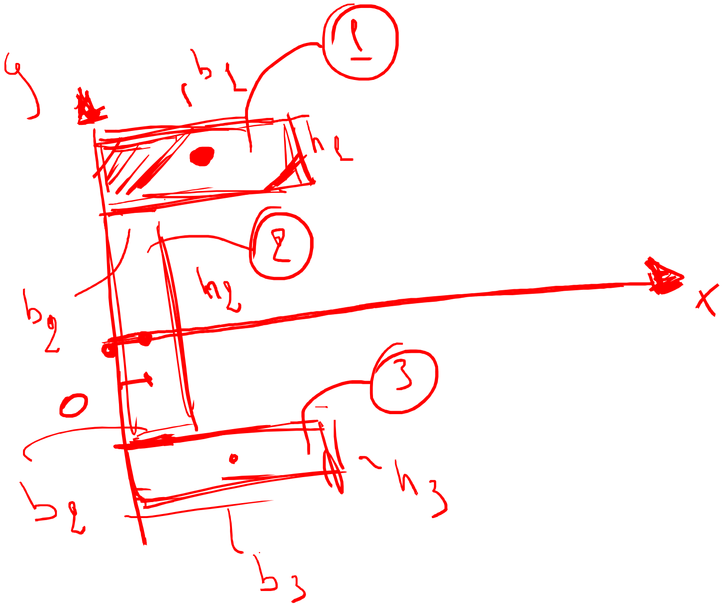
$$= \int_0^{2\pi} \sin\varphi \cos\varphi \, d\varphi \int_0^R \rho^3 \, d\rho$$

$$= \underbrace{\int_0^{2\pi} \frac{\sin 2\varphi}{2}}_{0} \int_0^R \rho^3 \, d\rho$$

$\sin(2\varphi)$



$$\Rightarrow I_{xy} = 0$$



1. $g(g^x, g^y)$

2. $\bar{I}_x$

$\bar{I}_y$

$\bar{I}_{xy}$

3. $G_0 \Rightarrow$ sistema di riferimento principale.

• $S_{y_i} = A_i \cdot x_{g_i} = \int_A x\,dA$; $x_g = \dfrac{\int_A x\,dA}{A}$ $\ulcorner S_y$

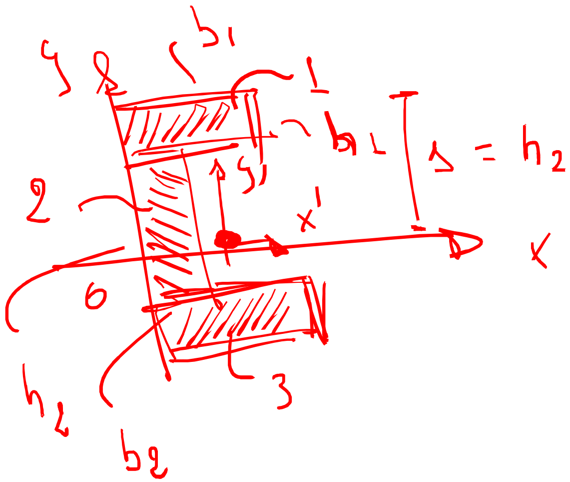
$\Rightarrow A \cdot x_g = \int_A x\,dA$

• $A \cdot x_g = S_y$

1. $g(g^x, g^y)$

Lo

$$g^x = \frac{S_y}{A} = \frac{S_{y1}+S_{y2}+S_{y3}}{A} = \left(\overbrace{(b_1h_1)}^{A_1}\overbrace{\frac{b_1}{2}}^{x_{g1}} + (b_2h_2)\,b_2/2 + (b_3h_3)\frac{b_3}{2}\right)(b_1h_1+b_2h_2+b_3h_3)$$



$$s = h_2/2 + h_1/2$$

$$\frac{S_y}{A} = g^x$$

$$g = (g^x, 0)$$

2. $\bar{I}_x$

$\bar{I}_y$

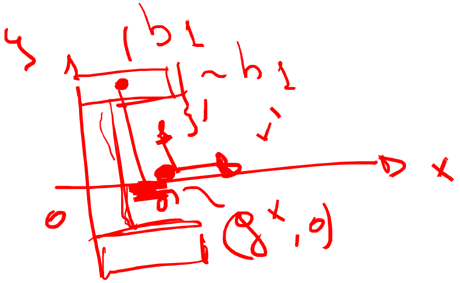
$\bar{I}_{xy}$

$$\bar{I}_x = \bar{I}_{x1} + \bar{I}_{x2} + \bar{I}_{x3}$$

$$= \left(\frac{b_1 h_1^3}{12} + \underbrace{(b_1 \cdot h_1)}_{A_1} \underbrace{\left(\frac{h_1+h_2}{2}\right)^2}_{s} \right) + \bar{I}^1_x$$

$$+ \left(\frac{b_2 h_2^3}{12} + (b_2 \cdot h_2)\, \cancel{0}^2 \right) + \bar{I}_{x2}$$

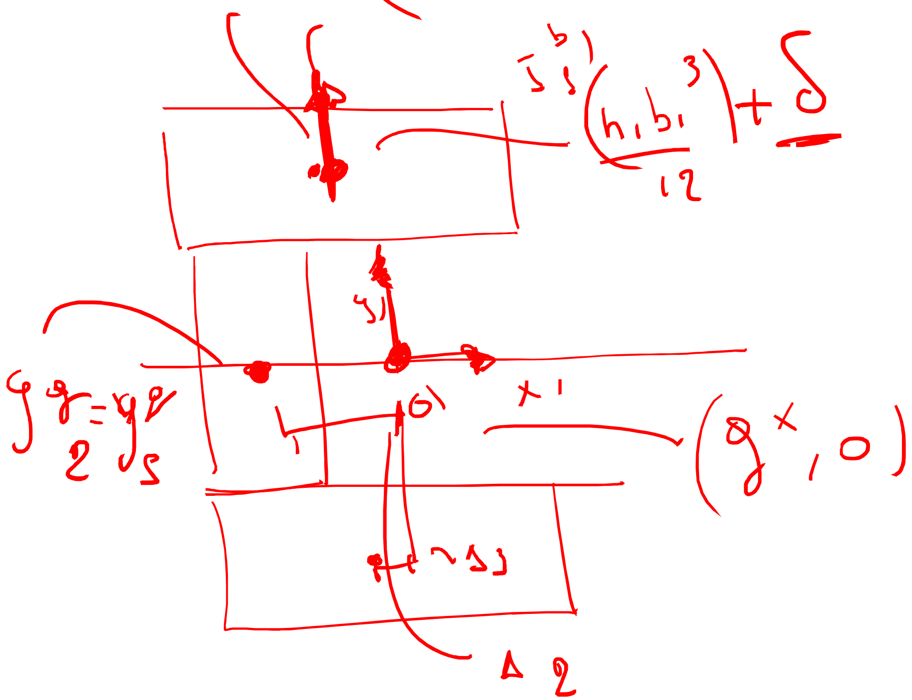
$$+ \left(\frac{b_3 h_3^3}{12} + (b_3 \cdot h_3) \left(\frac{h_3+h_2}{2}\right)^2 \right) \sim \bar{I}_{x3}$$



$$I_y = I_{y'1} + \underbrace{I_{y'2}}_{} + I_{y'3} \rightarrow I_{y1}$$

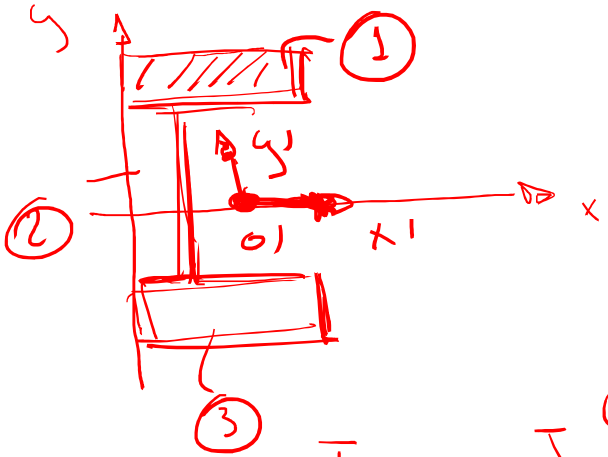
$$= \left(\underbrace{\frac{h_1 b_1^3}{12}}_{} + (h_1 b_1)\left(\frac{b_1}{2} - g^x\right)^2\right) + \quad I_{y2}$$

$$\left(b_1 l_2 l \; \frac{h_1 + h_2}{2}\right)$$



$$+ \frac{h_2 b_2^3}{12} + (h_2 b_2)(b_2/2 - g^x)^2 +$$

$$+ \frac{h_3 b_3^3}{12} + (h_3 b_3)(b_3/2 - g^x)^2$$



$$I_x \to I_{x'}$$

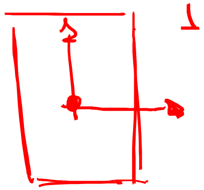
$$I_y \to I_{y'}$$

$$I_{xy'} \to I_{xy'}^{(1)} + I_{xy'}^{(2)} + I_{xy'}^{(3)}$$

$$I_{xy'} = \bar{I}_{xy}^{(0)} + A\,\Delta x\,\Delta y$$

$$I_{xy}^{(1)} = \cancel{\bar{I}_{xy}^{b\,(1)}} + \underline{A\,\Delta x\,\Delta y} = (b_1 h_1)\left(g^x - b_1/2\right)\left(0 - \frac{h_1+h_2}{2}\right)$$

0

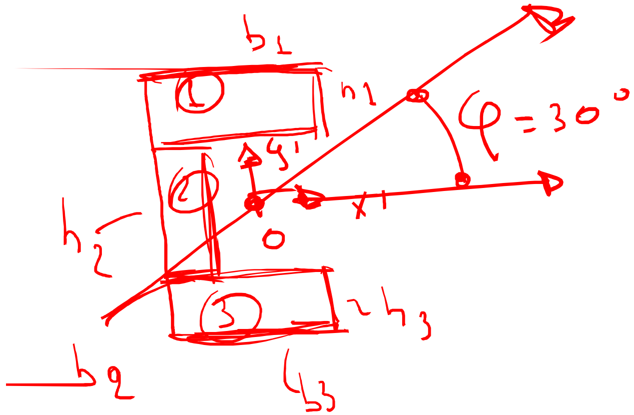


$$\bar{I}_{xy}^{b} = 0$$

$$I_{xy'}^{(2)} = 0 + A\,\cancel{\Delta x\,\Delta y} \to 0$$

(baricentro)

$$I_{xy'}^{(3)} = 0 + A\,\Delta x\,\Delta y = (b_3 \cdot h_3)\left(g^x - \frac{b_3}{2}\right)\left(+\frac{h_1+h_2}{2}\right)$$



$I_{x'}$

$I_{y'}$

$I_{x'y'}$

$$\begin{cases} b_1 = b_3 = 10\,mm \\ h_1 = h_3 = 2\,mm \\ b_2 = 2\,mm \\ h_2 = 8\,mm \end{cases}$$

$\varphi_0 \Rightarrow$ (ASSI PRINCIPALI DI INERZIA)

$$\tan(2\varphi_0) = \frac{2 I_{x'y'}}{I_{y'} - I_{x'}}$$

$$I_{xb} = \frac{b h^3}{12}$$